

Closing Today: 5A,5B,5C (6.5,7.1,7.2)

Next Wed: 6A,6B,6C (7.3,7.4,7.5)

Entry Task: Evaluate

$$\int \sqrt{9 + x^2} dx$$

7.3 Trig. Substitution

Steps-by-step:

1. Substitute as directed below.
Simplify (should eliminate root)
Don't forget dx.
2. Use 7.2 methods for trig integrals.
3. Draw a triangle and return to x.

CASE	SUBSTITUTION
$a^2 - x^2$	$x = a \sin(\theta)$
$a^2 + x^2$	$x = a \tan(\theta)$
$x^2 - a^2$	$x = a \sec(\theta)$

Example:

$$\int \frac{\sqrt{x^2 - 16}}{x} dx$$

Important Application

Area under a circle

$$\int \sqrt{4 - x^2} dx$$

Perfect Squares

$$(x + 3)^2 =$$

$$(x - 5)^2 + 2 =$$

Example:

$$\int \frac{1}{\sqrt{4x^2 - 32x + 100}} dx$$

Completing the Square

$$x^2 + 16x =$$

$$3x^2 - 12x =$$

Given $\sqrt{ax^2 + bx + c}$

complete square!

- *Factor out "a"*
- *Add/subtract half-middle squared*

Example:

$$\int \frac{x}{\sqrt{16 - 6x - x^2}} dx$$

7.4 Preview (Partial Fractions)

Goal: General method for rational functions.

$$\int \frac{x^3 + 4x - 4}{x^2(x^2 + 4)} dx = \int \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2 + 4} dx$$

Motivation:

$$\int \frac{x^3 + 4x - 4}{x^2(x^2 + 4)} dx = ??$$

We will learn to break up the fraction into:

$$\frac{x^3 + 4x - 4}{x^2(x^2 + 4)} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2 + 4}$$

Then we integrate each part separately.

Partial Fraction Decomposition

Step 0: Is the fraction *reduced*?

We mean the highest power on top is strictly smaller than the highest power on bottom.

If yes, move to step 1.

If not, divide, then move to step 1.

Example:

$$\int \frac{x^2 + x}{x + 3} dx$$

A fraction skill:

Reduce the “improper” fraction into a simplified mixed fraction:

$$\frac{576}{11} = ? + \frac{?}{11}$$

Partial Fractions Method Summary

Step 0: Reduce (if needed), see last page.

Step 1: Factor Denominator.

Write out decomposition below:

i) Distinct Linear:

$$\frac{x^2 - 3}{x(x - 1)(x + 4)} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 4}$$

ii) Repeated Linear:

$$\frac{5+2x}{(x+3)(x-2)^3} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

iii) Irreducible Quadratic:

$$\frac{4x}{(x + 1)(x^2 + 9)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 9}$$

Step 2: Solve for A, B, C

Step 3: Integrate

All the integrals in this section look like these:

$$\int \frac{1}{2x + 5} dx = \frac{1}{2} \ln|2x + 5| + C$$

$$\int \frac{1}{(x - 4)^2} dx = -\frac{1}{x - 4} + C$$

$$\int \frac{1}{(x + 7)^3} dx = -\frac{1}{2} \frac{1}{(x + 7)^2} + C$$

$$\int \frac{1}{x^2 + 9} dx = \frac{1}{3} \tan^{-1} \left(\frac{x}{3} \right) + C$$

$$\int \frac{x}{x^2 + 9} dx = \frac{1}{2} \ln|x^2 + 9| + C$$

The method uses algebra to rewrite **any** rational function as a sum of the integrals like those above.